University Competition:
Symmetric or Asymmetric Quality Choices?*

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Abstract: In this paper we model competition between two publicly financed and identical universities deciding on quality and on admission standards. The education offered by the two universities is differentiated horizontally and vertically. If horizontal differentiation dominates, the Nash equilibrium is symmetric, and the two universities offer the same quality levels. If vertical differentiation dominates, the Nash equilibrium is asymmetric, and the high quality university attracts the better students. Symmetric and asymmetric equilibria may also coexist. We highlight the importance of three driving forces behind these results: a single crossing condition, the peer group effect, and the students' mobility costs. We also compare the monopoly and the duopoly case. The model we use is an extension of Del Rey’s [8] model.

Key words: (higher) education, research, competition, horizontal and vertical dominance, asymmetric equilibria
1 Introduction

We consider two publicly financed and identical universities, competing in terms of the quality of the teaching programs offered and in terms of the applied admission standards. Their funding is provided by the government, and consists of a lump sum subsidy plus a fixed allowance per student. Their payoff is specified as a weighted sum of teaching output and available research funds.

Each student is characterized by a geographical location, and by a level of innate ability. Given these two characteristics, students rank the two universities in order of their preference. This ranking depends on two critical considerations. First, there are mobility costs. Each student is located at a certain distance from each university, implying a mobility cost for each university. Students with different locations face different mobility costs. These costs give rise to horizontal differentiation. Secondly, universities also offer study programs of different quality levels. This gives rise to vertical product differentiation.

For our results it is important to consider domination of one type of differentiation by the other. If all students living sufficiently close to a particular university, prefer that university to the other, for all levels of ability, we say that horizontal differentiation dominates vertical differentiation (i.e. there is horizontal dominance). Conversely, if all students with a sufficiently high (low) ability level prefer the high (low) quality university, for any given location, then vertical differentiation dominates (i.e. there is vertical dominance).

We show that each of the two types of domination gives rise to a different type of equilibrium. If horizontal differentiation dominates, the equilibrium quality levels offered by the two universities and the admission standards applied by each university will be the same. The Nash equilibrium is symmetric. If, on the other hand, vertical product differentiation dominates, equilibria can occur in which the two universities offer different equilibrium quality levels, and apply different admission standards. The high quality university attracts the better students. The Nash equilibrium is asymmetric. This is remarkable since the two universities are ex ante identical.

This result is consistent with the literature dealing with the interplay between horizontal and vertical differentiation within the field of industrial organization theory (e.g. Anderson, de Palma and Thisse [1] and Irmen and Thisse [13]). A basic result in this literature can be formulated as fol-
allows: minimal differentiation is possible in one dimension, only if differentiation is sufficiently large in the other. Applied to our model, this means that minimal differentiation in quality (symmetric quality levels) is only possible when mobility costs are sufficiently large (when there is horizontal dominance).

Depending on the exact parameter values the following equilibrium configurations occur: one symmetric equilibrium, two asymmetric equilibria, one symmetric and two asymmetric equilibria, and no equilibrium.

There are three basic characteristics that drive our results. First, preferences of the students have to satisfy a single crossing property. In particular, a student’s effort required to obtain a degree of a certain quality decreases as the student’s ability increases, and, for a given level of ability, a student’s required effort increases as quality increases. This seems to be a very reasonable property of any student’s preferences. We show that in the absence of this property, there can be no vertical product differentiation in equilibrium. Secondly, there is the peer effect. The larger the average ability level of the student body, the smaller the teaching cost required to realize a degree of a given quality level. Without this effect, there does not exist an equilibrium in which the universities are vertically differentiated. Finally, as already noted, there are mobility costs. If students do not care about the geographical location of the universities, we do not find an equilibrium.

In the paper we will analyze the monopoly as well as the duopoly case. We show that an increase in competition – a move from monopoly to duopoly – always raises the average quality level of teaching.

The literature on university competition is very limited. A first important contribution was made by Del Rey [8]. The model we will use in our paper is very similar to Del Rey’s model. We also study a two stage game in which universities first decide on quality levels, and then on admission standards. However, in Del Rey’s model there is no vertical product differentiation in equilibrium. Only symmetric Nash equilibria occur. We extend her model by introducing the single crossing condition already mentioned and by using a different specification for a university’s teaching cost function. As we will show, this drastically changes the model.

A second important reference is De Fraja and Iossa [6]. One of their main results is that asymmetric equilibria can occur, provided mobility costs are not too high. In this paper we
generalize this result by linking the symmetry or asymmetry of the equilibria to the properties of horizontal and vertical dominance and not only to mobility costs. Moreover, as opposed to De Fraja and Iossa [6] we also show that a symmetric equilibrium and two asymmetric equilibria can coexist. We argue that this would not have been possible when we would have used a simple lump sum funding mechanism comparable to De Fraja and Iossa [6]. Finally, our model has a much more explicit structure (cost function of teaching, objective function of the university, more general government funding, . . . ) giving more insight into the nature of the possible equilibria. In our model the three driving characteristics (the single crossing property, the peer group effect and the mobility costs) are each represented by a parameter.

Another interesting contribution to the literature on competition between universities was made by Debande and Demeulemeester [5]. They investigate competition between a university and a non-university institute deciding on the quality and variety (theoretical or vocational) of their curriculum when tuition fees are set by the government. Although the setting is totally different, they reach a conclusion similar to ours: differentiation will be the largest on the dimension students value the most (i.e. quality or variety).

Finally, we mention the existence of papers dealing with competition between two for-profit universities, like Melnik and Shy [16], or between a private and a public university, like Del Rey and Romero [10] and Oliveira [18]. As the objectives and the strategic instruments of the universities in these papers are very different from those we assume for our universities, this literature is less relevant for our purposes.

The structure of the paper is as follows. In section 2 we describe the behavior of the students and the universities. Section 3 analyzes the case of a monopolistic university. In Section 4 we solve the duopoly case. Section 5 concludes.

2 The model

In this section we describe the basic ingredients of the model. We first specify the behavior of the students. We then analyze the decisions of the universities.
2.1 The students

Consider a unit mass of students. Students are characterized by their physical location $x$, and by their innate ability (or talent) level $a$. These two characteristics are assumed to be uniformly and independently distributed on $[0, 1] \times [0, 1]$. We want to describe how a student with characteristics $(x, a)$ chooses between two universities. The two universities differ in their fixed physical location, and they can choose the quality of their degrees. University 1 is physically located at $x = 0$, and university 2 at $x = 1$. Moreover, university 1 offers a degree of quality level $q_1$, while university 2 offers a degree of quality level $q_2$. We assume that a student with ability $a$ and physically located at a distance $x$ from university 1 and a distance $(1 - x)$ from university 2 enjoys the following utility levels from attending university 1 and 2, respectively,

$$u_1 = \xi + q_1 - \alpha(1 - a)q_1 - cx$$

(1)

$$u_2 = \xi + q_2 - \alpha(1 - a)q_2 - c(1 - x).$$

(2)

First, simply attending a university augments the student’s utility with the constant $\xi$. We assume that $\xi$ is high enough, so that the student always prefers attending a university to not attending. In other words, we assume that the participation constraint $u_i \geq 0$ is always satisfied. Second, a student incurs a mobility cost which is taken to be proportional to the distance between her own physical location and that of the university at which she enrolls. A student located at a distance $x$ from university 1 faces a mobility cost of $cx$ when attending university 1, and of $c(1 - x)$ when attending university 2. Third, the quality of the degree offered affects a student’s utility in two different ways. On the one hand, a student’s future wage premium due to university education is increasing in the quality level of the university chosen.\footnote{Chevalier and Conlon [4] find a wage premium of up to 6\% for males graduating from the most prestigious (highest quality based on different quality measures) universities in the UK. Moreover, they try to control for the fact that high-ability students tend to select a high-quality university. Similarly, Brewer et al. [3] conclude for the US that, even after correcting for selection into the type of university, prestigious private universities yield significantly higher earnings compared to public universities.} On the other hand, obtaining a degree at a higher quality university requires a higher investment of effort from the student. The effect of this effort cost is given by $\alpha(1 - a)q_1$, where $\alpha$ is a positive number. The required effort cost decreases...
with the ability $a$ of the student. We neglect discounting. The utility levels in (1) and (2) imply the following *single crossing property*

$$\frac{\partial^2 u_i}{\partial q_i \partial a} = \alpha > 0.$$  \hfill (3)

See Mirrlees [17] and Spence [20]. It means that the net gain from an increase in quality is always higher for a higher ability student. Or, equivalently, the marginal effort cost for a degree of a given quality is decreasing in a student’s ability level. Moreover, notice that only high ability students benefit from attending a higher quality university

$$\frac{\partial u_i}{\partial q_i} = 1 - \alpha(1 - a) > 0 \iff a > 1 - \frac{1}{\alpha}.$$  \hfill (4)

The reason for this can be found in the effort cost which is lower for higher ability students.

In the main exposition of this paper we will always assume that $a$ equals 2. This simplifies the analysis without affecting the main results of the paper. For more details we refer to Appendix 4.

When we insert $a = 2$ the utility levels in (1) and (2) reduce to

$$u_1 = \xi + (2a - 1)q_1 - cx$$  \hfill (5)

$$u_2 = \xi + (2a - 1)q_2 - c(1 - x).$$  \hfill (6)

We now analyze the students’ choices between the two universities. The students who are indifferent between studying at university 1 and 2 mark the boundary between the two universities’ markets. Setting $u_1$ equal to $u_2$ and solving for $x$ yields the market boundary, denoted $\hat{x}(a)$

$$\hat{x}(a) = \frac{(2a - 1)(q_1 - q_2) + c}{2c}.$$  \hfill (7)

Students with characteristics $(x, a)$ such that $x \leq \hat{x}(a)$, prefer university 1. Students for whom $x \geq \hat{x}(a)$ prefer university 2. For a given quality difference $q_1 - q_2$, equation (7) defines a straight line in the $(x, a)$-space. Students to the left of this line prefer university 1. The number of these students is denoted by $d_1$, the demand for university 1. Students to the right of this line prefer university 2. The number of these students is denoted $d_2$, the demand for university 2. Since we

\[\text{Hence, effort is not chosen by the student herself. The student chooses a university with a certain quality level and this implies the required effort.}\]
assume that students always prefer attending a university to not attending one, it follows that
\[ d_1 + d_2 = 1. \]

Whenever the universities offer different quality levels, we will call university 1 the high and
university 2 the low quality university, \( q_1 - q_2 \geq 0 \). Consequently, the market boundary given in
(7) has a positive slope.

The distance between a student’s physical location and the location of a university differentiates
the two universities horizontally. The quality difference between the two universities differentiates
them vertically. Comparable to Anderson, De Palma and Thisse [1] and Degryse [7] we distinguish
between two possible cases: horizontal and vertical dominance. On the one hand, if \( 0 \leq q_1 - q_2 \leq c \),
we say that there is horizontal dominance: both universities attract a positive market share for
all ability levels. The slope of the market boundary is then larger than one. See Figure 1. The
demands for university 1 and 2 become
\[
d_1 = \int_0^1 \hat{x}(a) da \tag{8}
\]
and
\[
d_2 = \int_0^1 (1 - \hat{x}(a)) da. \tag{9}
\]
On the other hand, if the inequality \( q_1 - q_2 \geq c \) holds, we say that there is vertical dominance: the
high quality university 1 obtains the entire market for high ability students, while the low quality
university 2 attracts all low ability students. The slope of the market boundary is smaller than one.

![Figure 1: Horizontal dominance](image-url)
See Figure 2. The demands for university 1 and 2 are given by

\[
d_1 = \frac{a}{\hat{a}} \int \hat{x}(a) \, da + \int \hat{a} \, da \tag{10}
\]

and

\[
d_2 = \int _0^{\hat{a}} da + \frac{a}{\hat{a}} \int (1 - \hat{x}(a)) da \tag{11}
\]

with

\[
\hat{a} = \frac{1}{2} - \frac{c}{2(q_1 - q_2)} \tag{12}
\]

and

\[
a = \frac{1}{2} + \frac{c}{2(q_1 - q_2)} \tag{13}
\]

A rather surprising result is now that, for all values of \(q_1 - q_2 \geq 0\) and of \(c\),

\[
d_1 = d_2 = \frac{1}{2} \tag{14}
\]

This is easily checked from (7)-(13). It follows that, independent of the quality difference and of the mobility cost, both universities’ demands always equal one half of the total student population. It is important to realize that this result stems from the fact that we set \(\alpha\) equal to 2. See Appendix 4.

Furthermore, although both universities attract exactly the same number of students, it is clear from Figure 1 and Figure 2 that a higher quality university always attracts a student body with a higher average ability level.
The case \( q_1 - q_2 \leq 0 \) can be treated very similarly. The market boundary (7) will then have a negative slope. Expressions for both universities’ demands can be obtained by changing the subindexes 1 and 2.

Finally, note that in case \( \alpha = 0 \) the utilities (1) and (2) reduce to those used by Del Rey [8]. See also Appendix 4. Market boundaries are then vertical lines, independent of ability. In other words, students with the same physical location always prefer to attend the same university.

### 2.2 The universities

In this section we first show how universities can use admission standards to determine the number and the average ability of their students. We then specify a cost function of teaching, and the funding system used by the government. After introducing the payoff function of a university, we can finally describe the complete game played by the two universities.

When the two universities have decided on their quality levels \( q_1 \) and \( q_2 \), students can rank them in order of their preferences. Universities can then select students by setting an admission standard. In particular, each university can determine the ability level \( a_i \) of the least able student it admits.\(^3\) If a student is admitted at her preferred university, she will surely enroll. If she is not admitted at her preferred university, but if she meets the admission standard of the other university, she enrolls there. If she does not meet the admission standard of any university, she leaves the market for university students. This mechanism determines the number of students enrolled at each university. These numbers are denoted by \( n_1 \) and \( n_2 \). Together these decisions on admission standards also determine the average ability of the students enrolled at each university. These average abilities are denoted by \( \bar{a}_1 \) and \( \bar{a}_2 \). It is straightforward that, given the other university’s admission standard, setting a higher admission standard reduces a university’s enrolments, but increases the average ability of its enrolled students. For the mathematical expressions of \( n_1 \), \( n_2 \), \( \bar{a}_1 \), and \( \bar{a}_2 \) we distinguish 6 possible cases. Assuming \( a_1 \geq a_2 \), Figure 3 illustrates three different admission policies. In case \( a_1 \leq a_2 \), Figure 4 illustrates the remaining three cases that need to be investigated.

\(^3\)For instance, by imposing an admission test or setting a minimal grade students need to have obtained in secondary school.
We now turn to the teaching cost $T_i$ of university $i$. We specify this cost as

$$T_i = (1 - \eta \bar{a}_i) n_i q_i^2$$  \hspace{1cm} (15)$$

with $0 < \eta < 1$. A university’s teaching cost is decreasing in the average ability $\bar{a}_i$ of its enrolled students: the higher the average ability of the students, the smaller the expenses required to attain a given quality of education. This is also known as the peer group effect: students perform better among abler students.\(^4\) The strength of this effect depends on the parameter $\eta$. The teaching cost function given in (15) in fact expresses that students are inputs in the production process of their own human capital (Rothshild and White [19]). We assume that the parameter $\eta$ is always strictly between zero and one. We will see later on that our results are largely driven by the presence of this peer group effect. In other words, we need $\eta > 0$. Furthermore, a university’s teaching cost

\(^4\)Theoretical considerations on peer group effects can be found in e.g. Epple and Romano [11] and Winston [22]. Empirical investigation on peer group effects in tertiary education is done by e.g. Betts and Morell [2] and Zimmerman [23].
increases with the number of enrolled students $n_i$ and with the quality $q_i$ provided. Remark that, as opposed to Del Rey [8], we assume that marginal costs are increasing in quality.

The budget constraint of a university is kept very simple. As in Del Rey [8], a university receives a budget from the government consisting of a lump sum amount $F$, and a per student allowance $s$. Funds can be used to finance the teaching activities or can be spent on research. Research funds are denoted by $R_i$. The budget constraint is given by

$$F + sn_i = T_i + R_i.$$  \hspace{1cm} (16)

In Appendix 5 we investigate the case where the per student allowance $s$ equals 0. In that special case we find results similar to the ones we develop later on in this paper, except for the fact that the symmetric and the two asymmetric equilibria can no longer coexist.

The specification of a public university’s objective function is not straightforward. De Fraja and Iossa [6] start their discussion of it by stating that universities are interested in the “prestige” of their institution. This prestige depends on the number of students, on the average ability of the student body, and on the expenditures on research. The specification of Del Rey [8] is consistent with this general statement. It is given by

$$U_i = n_iq_i + \gamma R_i.$$  \hspace{1cm} (17)

The term $n_iq_i$ measures the teaching output of the university. This output depends on the number of students obtaining a degree, and on the quality level of this degree. $R_i$ represents the funds available for research. The weight attached to the latter equals $\gamma$. The same functional form is also used by Kemnitz [14]. In our model we will also use this specification. From (15) and (16) it follows that

$$R_i = F + sn_i - (1 - \eta \tilde{\alpha}_i)n_iq_i^2.$$  \hspace{1cm} (18)

The university’s payoff function can finally be written as

$$U_i(q_1, q_2; a_1, a_2) = n_iq_i + \gamma \left[F + sn_i - (1 - \eta \tilde{\alpha}_i)n_iq_i^2\right].$$  \hspace{1cm} (19)

A clear weakness of this specification is that the research output of a university is measured by the size of the research budget. This neglects the quality aspect of the research. Moreover, there
may be economies of scope between teaching and research. A high quality of teaching will also benefit the quality of research, and vice versa.\textsuperscript{5} We can introduce economies of scope in our model as follows. The true size of the research funds, denoted by $\tilde{R}_i$, can be defined such that a fraction $\mu$ of it is covered by the “gross” teaching expenditures $T_i = (1 - \eta \bar{a}_i)n_i q_i^2$. The “net” teaching expenditures are $\hat{T}_i = (1 - \eta \bar{a}_i)n_i q_i^2 - \mu \tilde{R}_i$. From the budget constraint $F + s n_i = \hat{T}_i + \tilde{R}_i$, we then obtain the true research expenditures $\tilde{R}_i = \frac{1}{1-\mu} \left[ F + s n_i - (1 - \eta \bar{a}_i)n_i q_i^2 \right] = \frac{1}{1-\mu} R_i$. If $\mu < 1$, $\tilde{R}_i$ will exceed $R_i$. The payoff function (19) can then be written as

$$U_i(q_1, q_2; a_1, a_2) = n_i q_i + \gamma \hat{R}_i = n_i q_i + \tilde{\gamma} R_i$$

(20)

with $\tilde{\gamma} = \frac{\gamma}{1-\mu}$. As $0 < \mu < 1$ we must have that $\tilde{\gamma} > \gamma$. An increase in the economies of scope between teaching and research is then equivalent to an increase in $\mu$, and to an increase of the weight $\gamma$ given to research funds in (19). In what follows, however, we will always use the notation involving $\gamma$ and $R_i$. When interpreting the results one can easily shift to $\tilde{\gamma}$ and $\tilde{R}_i$.

The complete game can now be specified as follows. In the first stage of the game the universities simultaneously decide on their quality levels $q_1$ and $q_2$. Students observe these levels, and rank the two universities. This was shown in Figure 1 and Figure 2. In the second stage of the game the two universities observe the quality levels $q_1$ and $q_2$, and they simultaneously decide on their admission standards $a_1$ and $a_2$. These decisions determine the numbers of enrolled students $n_1$ and $n_2$, and the average abilities of these students $\bar{a}_1$ and $\bar{a}_2$. See Figure 3 and Figure 4. The teaching outputs $n_1 q_1$ and $n_2 q_2$ are then determined. Knowing $q_i$, $n_i$ and $\bar{a}_i$, each university calculates its teaching cost $T_i$. Subtracting this cost from the total government subsidy $F + s n_i$ yields the available research funds $R_i$.

We will solve this game by backwards induction. However, before doing so, we first study the case in which one university has a monopoly. This is an important benchmark case.

\textsuperscript{5}In the current context, economies of scope exist if there are cost efficiencies to be gained by jointly producing teaching and research output, rather than producing each of them separately. Empirical confirmation of economies of scope between undergraduate teaching, graduate teaching and research can be found in Hashimoto and Cohn [12] and Koshal and Koshal [15].
3 The monopoly case

In this section we consider the case where there is only one university. As we assumed that all students want to attend a university ($u_i \geq 0$), the demand for the single university equals the total student population, independent of the quality of its teaching. If the monopolistic university fixes an admission standard $a_m$, all students who meet this standard, i.e. those with $a \geq a_m$, will actually enroll. Hence, the number of students $n_m$ equals $1 - a_m$ and their average ability level $\bar{a}_m$ becomes $\frac{1 + a_m}{2}$. The monopolistic university’s payoff function can now be written as

$$U_m(a_m; q_m) = n_m q_m + \gamma \left[ F + s n_m - (1 - \eta \bar{a}_m) n_m q_m^2 \right].$$

(21)

The university maximizes this function with respect to $a_m$ and $q_m$, subject to $0 \leq a_m \leq 1$. We will solve this problem in two steps. First, we maximize the payoff function with respect to $a_m$, subject to $0 \leq a_m \leq 1$, taking $q_m$ as given. This gives the optimal admission policy $a_m^*$ as a function of $q_m$. Second, using this optimal admission policy, we maximize the payoff function with respect to $q_m$. This way of solving the university’s problem will make it easier to compare the monopoly case with the sequential game played by the two universities, studied in Section 4.

3.1 The monopolist’s admission policy

For any given value of $q_m$, we first maximize (21) with respect to $a_m$, without taking into account the constraints. This yields us

$$a_m^* = \frac{-q_m - s \gamma + q_m^2 \gamma}{q_m^2 \gamma \eta} (\equiv \varphi(q_m))$$

(22)

It follows that $\varphi(q_m)$ is strictly increasing and strictly concave in $q_m$.

However, we know that the monopolist’s admission standard is constrained by $0 \leq a_m \leq 1$. It is easy to show that

$$0 \leq \varphi(q_m) \Leftrightarrow q_m \geq \frac{1 + \sqrt{1 + 4 \gamma^2 s}}{2 \gamma} (\equiv q^L)$$

(23)

and

$$\varphi(q_m) \leq 1 \Leftrightarrow q_m \leq \frac{1 + \sqrt{1 + 4 \gamma^2 s(1 - \eta)}}{2 \gamma (1 - \eta)} (\equiv q^H)$$

(24)
with \( q_L \leq q_H \) as long as \( \eta \geq 0 \). This allows us to conclude that the monopolist’s optimal admission standard is given by

\[
\begin{align*}
a_m^* &= 0 & \text{if } q_m \in [0, q^L], \\
a_m^* &= \frac{-q_m - s\gamma + q_m^2}{q_m^2 \gamma \eta} = \varphi(q_m) & \text{if } q_m \in [q^L, q^H] \text{ and } \quad (26) \\
a_m^* &= 1 & \text{if } q_m \in [q^H, \infty). \quad (27)
\end{align*}
\]

It follows that for low values of its first stage quality choice the monopolistic university will admit all students, i.e. there is no selection. For intermediate values of its quality choice the monopolist’s admission standard will be situated between zero and one. For very high quality levels the monopolistic university will decide to admit no student.

Notice that, in absence of the peer group effect, \( q^L \) equals \( q^H \). In other words, in that case the university either admits all students, or refuses all students.

### 3.2 The monopolist’s quality choice

For each of the three intervals \([0, q^L], [q^L, q^H]\) and \([q^H, \infty)\), the monopolist’s admission policy, is given in (25)-(27). We use these results to determine the corresponding values of \( n_m \) and \( a_m \). We then insert them into the payoff function (21) which becomes a function of \( q_m \) only. Finally, we maximize this function with respect to \( q_m \). In Appendix 1 we prove that the monopolist’s objective function (21) always attains its maximal value when its quality choice is situated in the interval \([0, q^L]\). From (25)-(27) it follows that the monopolist never finds it optimal to be selective. The optimal value of \( q_m \) is \( q_m^* = \frac{1}{\gamma(2-\eta)} < q^L \). Second order conditions are always satisfied. The following theorem easily follows.

**Theorem 1** A monopolistic university’s optimal quality level is given by

\[
q_m^* = \frac{1}{\gamma(2-\eta)} < q^L.
\]

Hence, the monopolistic university never finds it optimal to be selective, \( a_m^* = 0 \). Its optimal
teaching output, teaching expenditures and research funds are

\[ n^*_m q^*_m = \frac{1}{\gamma(2 - \eta)}, \quad T^*_m = \frac{1}{2\gamma^2(2 - \eta)} \quad \text{and} \quad R^*_m = F + s - \frac{1}{2\gamma^2(2 - \eta)}. \]

The university’s maximal payoff is equal to

\[ U^*_m = \gamma(F + s) + \frac{1}{2\gamma(2 - \eta)}. \]

From Theorem 1 it follows that the monopolist admits the total student population, independent of the per student allowance \( s \) it receives from the government. Consequently, when changing its funding mechanism, the government cannot affect the admission policy, nor the quality level chosen by the monopolistic university. Intuitively we would expect that a decrease in the per student allowance induces a university to raise its admission standard, and hence to provide higher quality. But, it seems that this gain in quality never outweighs the associated loss in students and attached funds, because the higher quality has no effect on the demand.\(^6\)

The following comparative statics results easily follow from Theorem 1. The higher the value of \( \eta \), i.e. the larger the peer group effect, the higher the monopolist’s quality choice, the more it spends on teaching and hence the lower the research funds. In absence of the peer group effect \( (\eta = 0) \) the monopolist’s quality choice equals \( \frac{1}{2\gamma} < q^L \), and hence the monopolist admits all students \( (a^*_m = 0) \).

Finally, the higher the monopolist’s preference for research \( \gamma \), the lower its quality choice, the less it spends on teaching and the higher the size of the research funds. In the limiting case, a pure research oriented university will enrol all students, cash all allowances attached to them, devote all funds to research and offer education of zero quality, \( \lim_{\gamma \to 1} q^*_m = 0 \) so that teaching costs are zero.

\(^6\)In a separate paper Del Rey [9] also investigates the monopoly benchmark for her model. She reaches two equilibrium types: full-time teaching and full-time research. In both cases all students are admitted by the monopolist. Both involve corner solutions in quality. As Del Rey [9] stresses, this stems from the separability and linearity in her teaching cost function. Since we use a non-linear and convex cost function we reach an interior solution for a monopolist’s optimal quality level. Nevertheless, our conclusion that changing the per-student allowance does not influence the behavior of a monopolist is entirely in line with the results of Del Rey [9].
4 The duopoly case

In this section we solve the duopoly game. In the first stage of this game the universities decide on quality levels \( q_1 \) and \( q_2 \). In the second stage there is, for each combination \((q_1, q_2)\), a corresponding subgame in which the universities decide on their admission standards \( a_1 \) and \( a_2 \). Using backwards induction we first solve these admission subgames. Afterwards, the Nash equilibria of these subgames are used to solve the complete game.

4.1 The admission subgame

Each combination of values \((q_1, q_2)\) defines an admission subgame. In this subgame, universities decide on their admission standards \( a_1 \) and \( a_2 \). By doing so they determine their numbers of enrolment \( n_1 \) and \( n_2 \), and the average ability levels of their students \( \tilde{a}_1 \) and \( \tilde{a}_2 \).

As a preliminary result, let us consider the case where university 1 and 2 simultaneously maximize their payoff function with respect to \( a_1 \) and \( a_2 \), subject to no constraint,

\[
\begin{align*}
\max_{a_1} U_1 &= n_1 q_1 + \gamma [F + s n_1 - (1 - \eta \tilde{a}_1) n_1 q_1^2] \\
\max_{a_2} U_2 &= n_2 q_2 + \gamma [F + s n_2 - (1 - \eta \tilde{a}_2) n_2 q_2^2].
\end{align*}
\]

In order to obtain the necessary first order conditions we first have to express both universities’ number of students \( n_1 \) and \( n_2 \), and their average abilities \( \tilde{a}_1 \) and \( \tilde{a}_2 \) as functions of the admission standards \( a_1 \) and \( a_2 \). As mentioned before, mathematically we distinguish 6 possible cases. See Figure 3 and Figure 4. In Appendix 2 we solve the case illustrated in the first graph of Figure 3.

Notice, however, that it can be proved that in all 6 cases the solution to (28) and (29) is given by

\[
a_i^* = \frac{-q_i - s \gamma + q_i^2 \gamma}{q_i^2 \gamma \eta} = \varphi(q_i), \quad i = 1, 2.
\]

This expression can usefully be compared with (22). We already noted that \( \varphi(q_i) \) is strictly increasing and strictly concave in \( q_i \). As we limit our analysis to the case where \( q_1 \geq q_2 \), it follows that \( \varphi(q_1) \geq \varphi(q_2) \). Consequently, for this unconstrained case, we find that a higher quality university sets a higher admission standard. However, the choice of the optimal value of \( a_i \) can be subject to two types of constraints.
First, let us consider the case of horizontal dominance \((q_1 - q_2 \leq c)\). If e.g. we take a look at the first graph of Figure 3 we see that the values of \(a_1\) and \(a_2\) are constrained by \(0 \leq a_i \leq 1\). Taking these constraints into account, we can again check that

\[
0 \leq \varphi(q_i) \Leftrightarrow q_i \geq \frac{1 + \sqrt{1 + 4\gamma^2 s}}{2\gamma}(\equiv q^L)
\]

and

\[
\varphi(q_i) \leq 1 \Leftrightarrow q_i \leq \frac{1 + \sqrt{1 + 4\gamma^2 s(1 - \eta)}}{2\gamma(1 - \eta)}(\equiv q^H)
\]

so that university \(i\)'s optimal admission standard is given by

\[
a_i^* = 0 \quad \text{if} \quad q_i \in [0, q^L],
\]

\[
a_i^* = \varphi(q_i) \quad \text{if} \quad q_i \in [q^L, q^H] \quad \text{and}
\]

\[
a_i^* = 1 \quad \text{if} \quad q_i \in [q^H, \infty).
\]

with \(i = 1, 2\). We conclude that in this case of horizontal dominance, university \(i\)'s admission policy is the same as the admission policy of a monopolist given in (25)-(27).

Second, we consider the case of vertical dominance \((q_1 - q_2 \geq c)\). On e.g. the second graph of Figure 3 we see that, assuming \(a_1 \geq a_2\), it makes no sense for the high quality university 1 to select an admission standard \(a_1\) below \(\hat{a}\). In other words, the high quality university 1 is constrained by its own demand. Hence, if

\[
\varphi(q_1) \leq \hat{a},
\]

which is equivalent to

\[
q_2 \leq \frac{q_1 \left[ 2s\gamma + q_1^2 \gamma (\eta - 2) + q_1(2 - c\gamma\eta) \right]}{2q_1 + 2\gamma s + q_1^2 \gamma(\eta - 2)}(\equiv g(q_1)),
\]

university 1's optimal admission standard equals \(\hat{a}\). There is no similar constraint on \(a_2\). It is quite possible that \(a_2 \geq \hat{a}\) or that \(a_2 \geq 2\).

Figure 5 now shows all possible areas in the \((q_1, q_2)\)-space for which the resulting subgames have qualitatively different Nash equilibria. Areas 1-3 are characterized by horizontal dominance \((q_1 - q_2 \leq c)\), the areas 4-8 by vertical dominance \((q_1 - q_2 \geq c)\). In areas 4-6, inequality (37) holds. All these results are summarized in the following Theorem 2.
Figure 5: Nash equilibria of the second stage as functions of first-stage quality choices

Theorem 2 Given \( q_1 \geq q_2 \), the unique Nash equilibria for the admission subgames are given by

\[
\begin{align*}
(a_1^* = 0, a_2^* = 0) & \quad \text{if} \quad (q_1, q_2) \in \text{area 1}, \\
(a_1^* = \varphi(q_1), a_2^* = 0) & \quad \text{if} \quad (q_1, q_2) \in \text{area 2 or 8}, \\
(a_1^* = \varphi(q_1), a_2^* = \varphi(q_2)) & \quad \text{if} \quad (q_1, q_2) \in \text{area 3 or 7}, \\
(a_1^* = \hat{a}, a_2^* = 0) & \quad \text{if} \quad (q_1, q_2) \in \text{area 4 or 5 and} \\
(a_1^* = \hat{a}, a_2^* = \varphi(q_2)) & \quad \text{if} \quad (q_1, q_2) \in \text{area 6}.
\end{align*}
\]

Remark that in all the equilibria for the subgame the high quality university 1 sets a higher admission standard than the low quality university 2, \( q_1 \geq q_2 \Rightarrow a_1^* \geq a_2^* \). Also note that all these Nash equilibria are based on (30). In some cases this value immediately applies in the Nash equilibrium. In other cases this value has to satisfy a constraint of the form \( 0 \leq a_i \leq 1 \), or of the form \( \hat{a} \leq a_1 \leq 1 \). Finally, we see that the admission standard selected by university 1 does not depend on the one selected by university 2, and vice versa. Hence, all the Nash equilibria of Theorem 2 are dominant strategy equilibria. See Appendix 2 for more details on this result.
4.2 Solving the first stage of the game

Now that we have solved the admission subgame for all possible quality levels, we move on to the first stage where the two universities simultaneously choose their quality levels. These quality choices determine the optimal admission standards in the second stage of the game, as given in Theorem 2.

Depending on the parameter values, the following Nash equilibrium configurations can appear:
(I) one symmetric equilibrium in which the quality levels of the two universities are equal, (II) two asymmetric equilibria in which one university provides higher quality education than the other, (III) one symmetric and two asymmetric equilibria, and (IV) no equilibrium. We analyze each of these possibilities. Before doing so, we make two important remarks.

First, when solving the first stage of the game, we always have to restrict ourselves to one of the areas in Figure 5. For each such area there is a corresponding optimal admission policy, and hence also a specific way in which \( n_i \) and \( a_i \) depend on \( q_i \). Using this dependence, we investigate whether there is a Nash equilibrium in quality levels. We then have to check whether this Nash equilibrium lies within the area. However, even if this is the case, it is not impossible that for one of the universities it is better to choose a quality level in a different area. In general, it is impossible to determine whether or not this is the case. Our Nash equilibria will therefore only be local equilibria. In numerical calculations it is possible to determine whether a university desires to switch to another area or not.

Second, we analyzed each area illustrated in Figure 5 with the method described above. Depending on the parameter values we found equilibria in areas 1, 4 or 5. Numerical computations suggest that there are no equilibria situated in the other areas. Unfortunately, we are not able to prove this in general.

4.2.1 Symmetric quality choices

In this section we investigate the case of horizontal dominance. From Theorem 2 we know that if both universities’ quality choices are low, and if the quality difference between them does not exceed the mobility cost, both will admit as many students as possible. More specifically, if \((q_1, q_2) \in \text{area}\)
the equilibrium admission policy equals \((a_1^* = 0, a_2^* = 0)\). The numbers of enrolment and the average abilities of both universities' student bodies are then given by

\[ n_1 = \int_0^1 \hat{x}(a) da, \quad \bar{a}_1 = \frac{1}{n_1} \left[ \int_0^1 a(\hat{x}(a)) da \right], \tag{38} \]

\[ n_2 = \int_0^1 (1 - \hat{x}(a)) da \quad \text{and} \quad \bar{a}_2 = \frac{1}{n_2} \left[ \int_0^1 a(1 - \hat{x}(a)) da \right]. \tag{39} \]

We insert these expressions into the payoff functions of the two universities, and we maximize each payoff function with respect to the quality level of that university. The resulting system of first order conditions gives the following unique and symmetric solution

\[ q_1^* = q_2^* = \frac{3c\gamma(2 - \eta) - \sqrt{3c\gamma(3c\gamma(-2 + \eta)^2 - 2\eta)}}{\gamma\eta} \tag{40} \]

Second order conditions are satisfied if and only if

\[ c > \frac{8\eta}{9\gamma(-2 + \eta)^2}. \tag{41} \]

Local stability of the equilibrium requires that

\[ \frac{\partial^2 U_1^*}{\partial q_1 \partial q_2} \frac{\partial^2 U_2^*}{\partial q_1 \partial q_2} > \frac{\partial^2 U_1^*}{\partial q_1 \partial q_2} \frac{\partial^2 U_2^*}{\partial q_2 \partial q_1}. \tag{42} \]

See Tirole [21], p. 324. This condition is satisfied if and only if

\[ c > \frac{6\eta}{5\gamma(-2 + \eta)^2}. \tag{43} \]

Notice that (41) is automatically satisfied if (43) is satisfied. We can conclude that the mobility cost \(c\) has to be sufficiently high to have a symmetric equilibrium in quality. This is similar to one of the main results of De Fraja and Iossa [6]. We still have to check ex post whether the equilibrium in quality given in (40) actually belongs to area 1. This is the case if and only if

\[ s > \frac{L^2 + 3c\gamma(-2 + \eta)(3c\gamma(-2 + \eta) + \eta) + L(6c\gamma(-2 + \eta) + \eta)}{\gamma^2\eta^2} \tag{44} \]

with \(L = \sqrt{3c\gamma(3c\gamma(-2 + \eta)^2 - 2\eta)}\). We see that the per student allowance \(s\) has to be sufficiently high to have a symmetric equilibrium in quality. In the paper of De Fraja and Iossa [6] this parameter equals zero.

\(^7\) Evaluated in the Nash equilibrium.
Now consider Theorem 3.

**Theorem 3** Assume that inequalities (43) and (44) hold. Then, there exists a symmetric, stable and local subgame perfect Nash equilibrium given by

\[ q_1^S = q_2^S = \frac{3c\gamma(2 - \eta) - \sqrt{3}c\gamma(3c\gamma(-2 + \eta)^2 - 2\eta)}{\gamma\eta}, \quad a_1^S = a_2^S = 0. \]

Numerical computations suggest that there are many reasonable sets of parameter values for which this equilibrium is global.

Two interesting properties of the equilibrium given in Theorem 3 are worth mentioning. First, within area 1 both universities’ reaction functions are downward sloping. See Figure 6. This follows from

\[ \frac{\partial^2 U_1}{\partial q_1 \partial q_2} = -\frac{q_1 \gamma \eta}{6c} < 0 \quad \text{and} \quad \frac{\partial^2 U_2}{\partial q_2 \partial q_1} = -\frac{q_2 \gamma \eta}{6c} < 0. \]

Hence, in this equilibrium the quality choices of the two universities are strategic substitutes: when university 1 raises its quality choice, university 2 will react by lowering its quality choice, and vice versa. Second, the equilibrium is symmetric, not only in the sense that \( q_1^S = q_2^S \), but also in the sense that \( n_1^S = n_2^S = \frac{1}{2}, \ a_1^S = a_2^S = \frac{1}{2}, \ T_1^S = T_2^S, \ R_1^S = R_2^S \) and \( U_1^S = U_2^S \). See Figure 7.

Let us summarize some comparative statics. Comparable to the monopoly case, we find that for the duopoly case the symmetric equilibrium quality level is independent of the financing scheme of
Figure 7: Symmetric division of students

the government. Again, we conclude that this stems from the fact that a change in quality has no effect on the size of a university’s demand. However, from (44) it follows that the existence itself of this equilibrium depends on the per student allowance \( s \). An increase in \( s \) dissuades a university to be selective. Hence, this allowance has to be high enough to ensure that the universities’ optimal admission standards equal zero. Again in line with the monopoly case, an increase in the peer group effect \( \eta \) raises the equilibrium quality level. From (40) it is not immediately clear what happens when we neglect the peer group effect \( (\eta = 0) \). It is easy to show, however, that in that case both universities set a zero admission standard and provide education with a quality level equal to \( \frac{1}{2} \).

An increase in the marginal utility of research \( \gamma \) decreases the equilibrium quality level. If both universities are entirely focused on maximizing research funds, both will enrol as much students as possible, cash all subsidies attached to these students, devote all funds to research and offer education of zero quality, \( \lim_{\gamma \to \infty} q_m^* = 0 \), so that teaching costs are zero.

Finally, we conclude that an increase in the mobility cost \( c \) leads to a reduction in the symmetric quality level

\[
\frac{\partial q_i^S}{\partial c} < 0 \quad i = 1, 2. \tag{46}
\]

This stems from the fact that, knowing that both universities admit as many students as possible \( (a_1^* = 0, a_2^* = 0) \), both always attract \( \frac{1}{2} \) of the total student population, independent of \( q_1 - q_2 \) and of \( c \). Hence, a change in the parameter \( c \) only influences the average ability of the student body.

\(^8\)In Appendix 4 we show that this is no longer the case when \( \alpha \) differs from 2.
attracted to each university. Now we find that
\[
\frac{\partial \bar{a}_i}{\partial q_i} = \frac{-1}{6c^2} < 0 \quad i = 1, 2.
\] (47)

This means that as the mobility cost increases, the positive effect of an increase in quality on the average ability of a university’s student body \( \frac{\partial \bar{a}_i}{\partial q_i} > 0 \) becomes smaller and smaller. Consequently, it follows that a university will lower its quality choice as the mobility cost increases. In the limiting case where the mobility costs go to infinity, the symmetric quality level for the duopoly case reduces to the quality choice of a monopolistic university, as given in Theorem 1.9 Hence, the quality offered by the duopolists always exceeds the quality offered by a monopolistic university, \( q^{*S} > q^{*m} \).

### 4.2.2 Asymmetric quality choices

In this section we investigate the case of vertical dominance. From Theorem 2 we know that if both universities’ quality choices are low, and if the quality difference between them exceeds the mobility cost, both will admit as many students as possible. However, in this case the highest quality university 1 will never attract students with an ability level below \( \bar{a} \). More specifically, if \((q_1, q_2) \in \text{area 4 or 5}\), the universities’ admission levels are given by \( (a_1^* = \bar{a}, a_2^* = 0) \). The student numbers and the average abilities are therefore equal to

\[
\begin{align*}
n_1 &= \int_{\bar{a}}^{\bar{a}} \hat{x}(a) da + \int_{\bar{a}}^{1} da, \quad \bar{a}_1 = \int_{\bar{a}}^{\bar{a}} a \hat{x}(a) da + \int_{\bar{a}}^{1} ada, \quad (48) \\
n_2 &= \int_{0}^{\bar{a}} da + \int_{\bar{a}}^{1} (1 - \hat{x}(a)) da \quad \text{and} \quad \bar{a}_2 = \int_{0}^{\bar{a}} ada + \int_{\bar{a}}^{1} a(1 - \hat{x}(a)) da. \quad (49)
\end{align*}
\]

We insert these expressions into the payoff functions of the universities. Maximizing each payoff function with respect to the quality level of that university yields two first order conditions. This system of equations has the following unique asymmetric solution

\[
q_1^{*A} = \frac{K}{12 \gamma^2 (4 - 3 \eta)}, \quad q_2^{*A} = \frac{K}{12 \gamma^2 (4 - \eta)}
\] (50)

9If the mobility costs go to infinity, students simply attend the university closest to their own physical location, \( \lim_{c \to \infty} (\hat{x}(a)) = \frac{1}{2} \).
with \( K = 12\eta^2 + \sqrt{6}\eta^2(24\eta^2 + c^2\gamma^2(16 - 16\eta + 3\eta^2)^2) \). Second order conditions require that

\[
c < \frac{8\sqrt{4 - \frac{2\eta}{3}\eta^{3/2}}}{\gamma(4 - 3\eta)^2(-4 + \eta)}.
\] (51)

The equilibrium is locally stable because the following condition is always satisfied \(^{10}\)

\[
\frac{\partial^2 U_1^*}{\partial q_1^2} \frac{\partial^2 U_2^*}{\partial q_2^2} > \frac{\partial^2 U_1^*}{\partial q_1 \partial q_2} \frac{\partial^2 U_2^*}{\partial q_2 \partial q_1}.
\] (52)

Finally, we have to check ex post whether (50) belongs to area 4 or 5. This requires simultaneous satisfaction of the following conditions

\[
c < \frac{24\eta}{5\gamma(-4 + \eta)(-4 + 3\eta)}, \] (53)

\[
s > \frac{K[(2 - \eta) + 6\eta^2(4 - c\gamma(4 - \eta))(-4 + 3\eta)]}{288\gamma^2(4 - 3\eta)^2\eta^4} \quad \text{and} \]

\[
s > \frac{K}{144\gamma^2(-4 + \eta)^2\eta^4}. \] (55)

We conclude from (51) and (53) that an asymmetric equilibrium in quality requires a low mobility cost \( c \). Again, this confirms the findings of De Fraja and Iossa [6]. Moreover, from (54) and (55) it follows that our asymmetric equilibrium in quality is only feasible for sufficiently high values of the per student allowance \( s \).

Now consider Theorem 4.

**Theorem 4** Assume conditions (51) and (53)-(55) hold. Then there exists an asymmetric, stable and local subgame perfect Nash equilibrium in which

\[
q_1^* = \frac{K}{12\gamma\eta^2(4 - 3\eta)}, \quad q_2^* = \frac{K}{12\gamma\eta^2(4 - \eta)}, \quad a_1^* = 0, \quad a_2^* = 0.
\]

Of course, since the two universities are identical it must be that there also exists a similar asymmetric equilibrium in which university 2 offers a higher quality level than university 1

\[
q_1^* = \frac{K}{12\gamma\eta^2(4 - \eta)}, \quad q_2^* = \frac{K}{12\gamma\eta^2(4 - 3\eta)}.
\] (56)

*Evaluated in the Nash equilibrium.*
In Appendix 3 we show that in absence of mobility costs \( c = 0 \) university 2 will have an incentive to jump to another area in the \((q_1, q_2)\)-space. In other words, in that case the asymmetric equilibrium only holds locally. The same reasoning applies for sufficiently small values of \( c > 0 \). Numerical experimentation confirms that there do exist values of \( c \) which satisfy (51) and (53), and for which university 2 has no incentive to jump to another area, so that the asymmetric equilibrium of Theorem 4 is global. Unfortunately, we can not determine an explicit threshold value of \( c \).

Let us discuss three interesting features of the equilibrium given in Theorem 4. First, we find that within area 4 and 5 the reaction function of the high quality university 1 is upward sloping while the one of the low quality university 2 is downward sloping,

\[
\frac{\partial^2 U_1}{\partial q_1 \partial q_2} = \frac{c^2 q_1 (q_1 + 2q_2) \gamma \eta}{12(q_1 - q_2)^4} > 0 \quad \text{and} \quad \frac{\partial^2 U_2}{\partial q_2 \partial q_1} = -\frac{c^2 q_2 (q_2 + 2q_1) \gamma \eta}{12(q_1 - q_2)^4} < 0.
\]

(57)

This means that an increase in \( q_2 \) induces university 1 to increase \( q_1 \), while an increase in \( q_1 \) induces university 2 to decrease \( q_2 \). See Figure 8.\(^{11}\) Second, at the equilibrium described in Theorem 4 the universities equally share the market in the sense that \( n_{A1} = n_{A2} = \frac{1}{2} \). Notice that this results from setting \( \alpha \) equal to 2. See Appendix 4. Third, university 1 offers a higher quality level, and hence attracts a student body with a higher average ability compared to university 2. See Figure 9. Despite the higher average ability level of university 1’s student body, it spends more on teaching than university 2, \( T_{1A} > T_{2A} \). Hence, university 1 has less research funds \( R_{1A} < R_{2A} \). Under conditions (51)-(55), the equilibrium payoff of university 1 always exceeds the equilibrium payoff of university 2

\[
U_{1A} = n_{1A} q_{1A} + \gamma R_{1A} > U_{2A} = n_{2A} q_{2A} + \gamma R_{2A}.
\]

(58)

This results from setting \( \alpha \) equal to 2. See Appendix 4.

Next we summarize some comparative statics. Similar to the previous case, the universities’ quality choices at the asymmetric equilibrium in Theorem 4 are not influenced by the per student

\(^{11}\)In this Figure we see that, for certain parameter values, the reaction functions are not continuous. As \( q_2 \) starts to increase from 0, university 1 reacts by increasing \( q_1 \). However, when \( q_2 \) reaches a certain level it is no longer a best reply for university 1 to stay on increasing \( q_1 \) (probably because the marginal cost of quality is increasing). Its best reply is now situated in another area, i.e. that in which it becomes the lowest quality university. Of course, the same reasoning applies to the reaction function of university 2.
allowance $s$. This value of $s$, however, does appear in (54) and (55). In line with previous results, an increase in $\gamma$ decreases the asymmetric quality levels, while an increase in $\eta$ increases the asymmetric quality levels. Remark that when we neglect the peer group effect ($\eta = 0$), there does not exist an asymmetric equilibrium.

Consider the following two interesting notes on the role of the parameter $c$ in the asymmetric equilibrium. First, as opposed to (46), an increase in the mobility cost $c$ now raises the equilibrium quality levels

$$\frac{\partial q_i^*}{\partial c} > 0 \quad i = 1, 2.$$  \hspace{1cm} (59)

Again, knowing that both universities admit as many students as possible ($a_1^* = \hat{a}$, $a_2^* = 0$), both always attract $\frac{1}{2}$ of the total student population, independent of $q_1 - q_2$ and of $c$. Hence, a change in the parameter $c$ only influences the average ability of the student body attracted to each university. As opposed to (47) we now find that

$$\frac{\partial \hat{a}_i}{\partial q_i \partial c} = \frac{c}{3(q_1 - q_2)^3} > 0 \quad i = 1, 2.$$  \hspace{1cm} (60)
This means that as the mobility cost increases, the positive effect of an increase in quality on the average ability of a university’s student body ($\frac{\partial a_i}{\partial q_i} > 0$) becomes larger and larger. It follows that a university will increase the quality provided as the mobility cost increases. Second, we find that the degree of quality differentiation in the asymmetric equilibrium tends to increase with the mobility cost $c$

$$\frac{\partial(q_1^A - q_2^A)}{\partial c} > 0.$$  

(61)

In fact, this states that as the horizontal differentiation (measured by the size of $c$) between the universities increases, the vertical differentiation (measured by $q_1^A - q_2^A$) increases as well. This may sound counterintuitive, but, remember that we need a low level of $c$ for the asymmetric equilibrium to hold (see (51) and (53)), while the symmetric equilibrium requires a high level of $c$ (see (43)). Hence, as $c$ increases we move from the asymmetric to the symmetric equilibrium in quality, meaning that an increase in horizontal differentiation leads to a decrease in vertical differentiation.

Finally, under conditions (51), (53)-(55), the quality provided by university 1 (2) is higher (lower) than the quality choice of a monopolistic university, $q_2^A < q_m^* < q_1^A$. Moreover, the average teaching output in this duopoly case exceeds the teaching output produced by a monopolistic university, $n_1^A q_1^A + n_2^A q_2^A > n_m^* q_m^*$.

### 4.2.3 Symmetric and asymmetric equilibria – No equilibrium

In the previous two sections we derived the necessary conditions for the existence of a local symmetric equilibrium ((41)-(44)). We also determined the necessary conditions for the existence of two local asymmetric equilibria ((51)-(55)). Therefore, we know which parameter values lead to which type of local equilibria.

We now show that for certain parameter values both sets of conditions, (41)-(44) and (51)-(55), are simultaneously satisfied, so that the two types of local equilibria coexist. Moreover, it is also possible that no set of conditions is satisfied, so that there is no equilibrium in pure strategies. To

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12To be sure that these local equilibria are also global equilibria, we would again have to rely on numerical calculations.
simplify the analysis, we assume that the two universities care as much for teaching as for research \((\gamma = 1)\), and that the peer group effect is quite high \((\eta = 0.8)\). The remaining parameters are the mobility cost \(c\) and the per student allowance \(s\).

Within the \((c, s)\)-space we identify four regions, given on Figure 10.\(^{13}\) In the region labeled I, conditions (51)-(55) are satisfied and we find two asymmetric equilibria as described in Theorem 4. We see that this region is characterized by a low level of the mobility cost. The symmetric equilibrium from Theorem 3 can be found in region II where the inequalities (41)-(44) hold. It follows that sufficiently high values of the mobility cost always lead to a symmetric equilibrium. In region III, conditions (41)-(44) as well as (51)-(55) are satisfied, and hence the symmetric equilibrium and the two asymmetric equilibria coexist. The symmetric equilibrium is characterized by horizontal dominance, the asymmetric equilibria are characterized by vertical dominance. In this region we have a moderate mobility cost and a relatively high per student allowance. Both universities’ reaction functions can be found in Figure 11. Remark that when we assume that the per student allowance equals zero, this region III disappears. In other words, in that case the symmetric equilibrium and the two asymmetric equilibria never coexist. See Appendix 5. Finally, in region IV neither of the two sets of conditions is satisfied. Hence, in this region we do not find an equilibrium in pure strategies. This last region is characterized by a moderate mobility cost and a low per student allowance. Figure 12 gives the reaction functions.

\(^{13}\)Remark that Figure 10 largely depends on the chosen parameter values. Other values of \(\gamma\) and \(\eta\) could yield a different picture.
5 Conclusion

We developed a duopoly model in which universities compete in the quality of their teaching and in their admission policies. University funding is provided by the government. More specifically, each university receives a lump sum amount and an amount depending on its number of students. A university’s teaching cost is decreasing in the average ability of its student body, increasing in its number of students and in its quality choice. Universities care for research funds as well as teaching output. Students are characterized by an ability level and a geographical location.

The model captures two dimensions of product differentiation. First, the mobility costs students incur when travelling to a university differentiates the universities horizontally. Second, the quality difference between the universities differentiates them vertically.

The two main results of this paper can be summarized as follows. First, we found that both universities will provide the same quality and will adopt the same admission policy when horizontal differentiation dominates vertical differentiation. This requires a mobility cost $c$ which is
sufficiently high. Second, if vertical differentiation dominantes horizontal differentiation, the universities offer different quality levels and use different admission policies. This requires a mobility cost which is sufficiently low. The two results imply that as the mobility cost increases we move from the asymmetric to the symmetric equilibrium in quality. In other words, the degree of vertical differentiation tends to decrease when the degree of horizontal differentiation increases.

From our model it also follows that the average teaching output produced in a duopoly exceeds the teaching output produced by a monopolist. Moreover, we show that universities who entirely focus on research and hence neglect their teaching, adopt a free admission policy in order to cash as much subsidies as possible, provide zero quality, have the maximum of research funds and zero teaching costs. Finally, we find that if we would drop the assumption that more talented students allow a university to save on teaching costs (i.e. the peer group effect), the resulting equilibrium would always be symmetric.

Our paper is closely related to the work of Del Rey [8] and De Fraja and Iossa [6]. There are two important differences. First, as opposed to Del Rey [8], we included the assumption that a student’s preference for quality depends on her ability (i.e. the single crossing condition), and we used a different teaching cost function. Consequently, we found equilibria characterized by quality differentiation. Del Rey [8] only found symmetric equilibria. Second, as opposed to De Fraja and Iossa [6] we do not assume that all students necessarily prefer the highest quality university\footnote{This stems from the fact that in our model a student’s effort cost is increasing in quality and decreasing in her ability.}, we use a different university funding formula and we make a distinction between a university’s quality level and its admission standard. Although the setting clearly differs, we found the same correspondence between the size of the mobility cost and the type of equilibrium as De Fraja and Iossa [6] did. However, we generalized this result to a correspondence between horizontal and vertical dominance and the type of equilibrium. Moreover, De Fraja and Iossa [6] can not define a region of the parameter values in which the symmetric and asymmetric equilibria coexist. In Appendix 5 we show that this would also apply to our model if we would use their funding formula instead of ours.

Within the context of our model it might be interesting to look at the following two extensions.
First, we can ask ourselves which quality levels and admission standards maximize social welfare. Social welfare could e.g. be defined as the sum of the students’ utilities minus the costs of education (Del Rey and Romero [10] and Oliveira [18]). Second, we could include a participation constraint which could be binding for some types of students. Depending on their characteristics \((x, a)\) some students may not find it worthwhile to participate in university education. In this case, the demand for a university will depend on the quality offered and the mobility cost. This will surely influence the competition between universities.
Appendix 1: Optimal quality choice of a monopolist (proof of Theorem 1)

First, let us suppose that the optimal quality choice of a monopolistic university lies within $[0, q^L]$. We know that in this case the university is not selective; hence, its number of enrolled students equals its demand, $n_m = d_m = 1$. The average ability of these students equals one half, $a_m = \frac{1}{2}$. Substituting all this into the university’s payoff function and maximizing it with respect to $q_m$ yields $q^*_m = \frac{1}{\gamma(2-\eta)}$. Second order conditions are always satisfied. We still need to check whether this solution really lies within the interval $[0, q^L]$. This requires $s > \frac{-1+\eta}{\gamma(-1+\eta)^2}$. This condition always holds. Second, we investigate whether an optimal quality choice can be found within $[q^L, q^H]$. This would mean that the university adopts a selective admission policy $a^*_m$ equal to $\varphi(q_m)$. We insert $n_m = 1 - \varphi(q_m)$ and $a_m = \frac{1+\varphi(q_m)}{2}$ into the university’s objective function and maximize it with respect to its quality choice. However, we see that, in the relevant range, the university’s objective function is strictly decreasing in its quality choice,

$$\frac{\partial U_m}{\partial q_m} = \frac{(-s + q_m^2(-1+\eta))(q_m + s\gamma + q_m^2\gamma(-1+\eta))}{q_m^2\eta} < 0 \Leftrightarrow q_m < q^H. \quad (a1)$$

Finally, if we suppose that the university’s quality choice lies within $[q^H, \infty)$, its optimal admission standard $a^*_m$ equals 1. Consequently, no student is admitted and the university’s payoff function equals $\gamma F$, independent of its quality choice. We can conclude that the payoff of a monopolistic university always reaches a global maximum within the interval $[0, q^L]$ at a value of $q^*_m$ equal to $\frac{1}{\gamma(2-\eta)}$.

Appendix 2: The second stage of the game

In the second stage of the game both universities decide on their admission standards $a_1$ and $a_2$. In this appendix we mathematically work out the case where $a_1 \geq a_2$ and where there is horizontal dominance, illustrated in the first graph of Figure 3. First, we will show that in the second stage of the game both universities have dominant strategies. Second, we will explicitly solve for these strategies. The proof for the other 5 cases can be found in a very similar way.
Suppose \( q_1 - q_2 < c \) and \( a_1 \geq a_2 \). Then we know that

\[
n_1 = \int_{a_1}^{1} \hat{x}(a) da, \tag{a2}
\]

\[
\bar{a}_1 = \frac{1}{n_1} \left[ \int_{a_1}^{1} a(\hat{x}(a)) da \right] \tag{a3}
\]

and that

\[
n_2 = \int_{a_2}^{a_1} da + \int_{a_1}^{1} (1 - \hat{x}(a)) da, \tag{a4}
\]

\[
\bar{a}_2 = \frac{1}{n_2} \left[ \int_{a_2}^{a_1} ada + \int_{a_1}^{1} a(1 - \hat{x}(a)) da \right]. \tag{a5}
\]

We need (a2)-(a5), because in the second stage of the game the two universities simultaneously select an admission standard by solving

\[
\max_{a_1} U_1 = n_1 q_1 + \gamma \left[ F + s n_1 - (1 - \eta \bar{a}_1) n_1 q_1^2 \right] \tag{a6}
\]

\[
\max_{a_2} U_2 = n_2 q_2 + \gamma \left[ F + s n_2 - (1 - \eta \bar{a}_2) n_2 q_2^2 \right]. \tag{a7}
\]

When changing its own admission standard a university affects its student number and the average ability of its student body.\(^{15}\) Hence, it follows that

\[
\frac{\partial n_1}{\partial a_1} = -\hat{x}(a_1), \quad \frac{\partial n_1 \bar{a}_1}{\partial a_1} = -a_1 \hat{x}(a_1) \quad (= a_1 \frac{\partial n_1}{\partial a_1}) \tag{a8}
\]

and that

\[
\frac{\partial n_2}{\partial a_2} = -1, \quad \frac{\partial n_2 \bar{a}_2}{\partial a_2} = -a_2. \tag{a9}
\]

Of course, when a university changes its admission standard, this also affects the student number and the average ability of the rival university. However, what is important is that

\[
\frac{\partial n_1}{\partial a_2 \partial a_2} = 0, \quad \frac{\partial n_1 \bar{a}_1}{\partial a_2 \partial a_2} = 0 \tag{a10}
\]

and that

\[
\frac{\partial n_2}{\partial a_2 \partial a_1} = 0, \quad \frac{\partial n_2 \bar{a}_2}{\partial a_2 \partial a_1} = 0. \tag{a11}
\]

\(^{15}\)Due to the chosen specification of the cost function we are only interested in \( \frac{\partial n_1 \bar{a}_1}{\partial n_2} \), not in \( \frac{\partial n_1}{\partial n_2} \).
This means that the effect of a change in a university’s admission standard will not depend on the admission standard chosen by its competitor. Consequently, we can already conclude beforehand that both universities’ admission standards will involve dominant strategies.

Let us now explicitly solve (a6) and (a7). For university 1 we find that

$$\frac{\partial U_1}{\partial a_1} = q_1 \frac{\partial n_1}{\partial a_1} + \gamma s \frac{\partial n_1}{\partial a_1} - \gamma q_1^2 \frac{\partial n_1}{\partial a_1} + \gamma \eta q_1^2 \frac{\partial n_1}{\partial a_1}. \quad (a12)$$

We use (a8) to obtain the solution for university 1’s admission standard as follows

$$\frac{\partial U_1}{\partial a_1} = \frac{\partial n_1}{\partial a_1} [q_1 + \gamma s - \gamma q_1^2 + \gamma \eta q_1^2 a_1] \quad (a13)$$

$$\frac{\partial U_1}{\partial a_1} = 0 \Leftrightarrow (1) \frac{\partial n_1}{\partial a_1} = 0 \text{ or } (2) q_1 + \gamma s - \gamma q_1^2 + \gamma \eta q_1^2 a_1 = 0 \quad (a14)$$

$$q_1 = 0 \Leftrightarrow a_1 = 1 - \frac{c}{2(q_1 - q_2)} \quad (a15)$$

$$q_1 + \gamma s + \gamma q_1^2 \gamma \eta q_1^2 \quad (a16)$$

We know that when \( q_1 - q_2 < c \), as assumed at the beginning, (a15) will always be negative. Since a negative admission standard makes no sense, we can ignore this solution. Solution (a16) satisfies the constraints \( 0 \leq a_1 \leq 1 \) if and only if \( q^L \leq q_1 \leq q^H \), with \( q^L = \frac{1+\sqrt{1+4s^2}}{2\gamma} \) and \( q^H = \frac{1+\sqrt{1+4s^2(1-\eta)}}{2\gamma(1-\eta)} \).

To conclude, for the case of horizontal dominance, we have proven that university 1 will select the following admission standard

$$a_1^* = 0 \Leftrightarrow q_1 \leq q^L \quad (a17)$$

$$a_1^* = \frac{-q_1 - \gamma s + \gamma q_1^2}{\gamma \eta q_1^2} \Leftrightarrow q^L \leq q_1 \leq q^H \quad (a18)$$

$$a_1^* = 1 \Leftrightarrow q_1 \geq q^H. \quad (a19)$$

As expected, \( a_1^* \) is independent of \( a_2 \).

For university 2 we find that

$$\frac{\partial U_2}{\partial a_2} = q_2 \frac{\partial n_2}{\partial a_2} + \gamma s \frac{\partial n_2}{\partial a_2} - \gamma q_2^2 \frac{\partial n_2}{\partial a_2} + \gamma \eta q_2^2 \frac{\partial n_2}{\partial a_2} \quad (a20)$$

We use (a9) to obtain the solution for university 2’s admission standard as follows

$$\frac{\partial U_2}{\partial a_2} = \frac{\partial n_2}{\partial a_2} [q_2 + \gamma s - \gamma q_2^2] + \gamma \eta q_2^2 \frac{\partial n_2}{\partial a_2} \quad (a21)$$

$$\frac{\partial U_2}{\partial a_2} = 0 \Leftrightarrow -q_2 - \gamma s + \gamma q_2^2 - \gamma \eta q_2^2 a_2 = 0 \Leftrightarrow a_2 = \frac{-q_2 - \gamma s + \gamma q_2^2}{\gamma \eta q_2^2} \quad (a22)$$
This solution satisfies $0 \leq a_2 \leq 1$ if and only if $q^L \leq q_2 \leq q^H$. We can conclude that, for the case of horizontal dominance, we have proven that university 2 will select the following admission standard

$$a_2^* = 0 \Leftrightarrow q_2 \leq q^L\quad (a23)$$

$$a_2^* = \frac{-q_2 + \gamma s + \gamma q_2^2}{\gamma q_2^2} \Leftrightarrow q^L \leq q_2 \leq q^H\quad (a24)$$

$$a_2^* = 1 \Leftrightarrow q_2 \geq q^H.\quad (a25)$$

Again, we see that $a_2^*$ does not depend on $a_1$.

**Appendix 3: No mobility costs ($c = 0$)**

We assume that $q_1 > q_2$ and that students incur no cost when travelling from their own geographical location to that of the university they attend. Then it follows that university 1 will attract all students with an ability level above one half ($\tilde{a} = a = \frac{1}{2}$), while the other students prefer to attend university 2. Furthermore, if $a_1 \geq \frac{1}{2}$ and $a_1 \geq a_2$, we find

$$n_1 = 1 - a_1, \quad \tilde{a}_1 = \frac{1 + a_1}{2},\quad (a26)$$

$$n_2 = a_1 - a_2 \text{ and } \tilde{a}_2 = \frac{a_1 + a_2}{2}.\quad (a27)$$

See Figure 13.

Maximizing university $i$’s payoff function (19) with respect to $a_i$, subject to no constraint on $a_i$ again yields $a_i^* = \varphi(q_i)$ (see 30). Notice that every possible quality combination $(q_1, q_2)$ is characterized by *vertical dominance* since $q_1 - q_2 \geq c = 0$. Hence, university 1’s admission standard is subject to $\tilde{a} \leq a_1 \leq 1$. This implies that if

$$\varphi(q_1) \leq \tilde{a},\quad (a28)$$

![Figure 13: $c = 0$](image)
which is equivalent to
\[ q_1 \leq \frac{1 + \sqrt{1 - 2s\gamma^2(-2 + \eta)}}{\gamma(2 - \eta)} (\equiv q^M), \] (a.29)

university 1’s optimal admission standard equals \( \hat{a} \). The constraints on university 2’s admission standard have no other consequences than those described in the paper. Hence, the two values \( q^L \) and \( q^H \) are as defined in (31) and (32).

Below you find the reformulation of Theorem 2 for the case \( c = 0 \).

Given \( q_1 \geq q_2 \) and \( c = 0 \), the unique Nash equilibria for the admission subgames become
\[
\begin{align*}
(a_1^* = \varphi(q_1), a_2^* = \varphi(q_2)) & \quad \text{if } q_1 \in [q^M, q^H] \text{ and } q_2 \in [q^L, q^H], \\
(a_1^* = \varphi(q_1), a_2^* = 0) & \quad \text{if } q_1 \in [q^M, q^H] \text{ and } q_2 \in [0, q^L], \\
(a_1^* = \hat{a}, a_2^* = \varphi(q_2)) & \quad \text{if } q_1 \in [0, q^M] \text{ and } q_2 \in [q^L, q^H], \\
(a_1^* = \hat{a}, a_2^* = 0) & \quad \text{if } q_1 \in [0, q^M] \text{ and } q_2 \in [0, q^L].
\end{align*}
\]

So far for the subgame. For the first stage of the game we will see that, assuming \( q_1 \geq q_2 \) and \( c = 0 \), the resulting subgame perfect Nash equilibrium is asymmetric in quality. But, we will prove that the low quality university 2 always has an incentive to deviate to \( q_1^A + \epsilon \).

If \( q_1 \in [q^M, q^H] \) and \( q_2 \in [q^L, q^H] \), the optimal admission policy equals \( (a_1^* = \varphi(q_1), a_2^* = \varphi(q_2)) \). Using this information we simultaneously maximize both universities’ payoff functions with respect to their quality level. We find that university 1’s payoff function is strictly decreasing in its quality level. Notice the similarity with a monopolist. This allows us to conclude that there is no Nash equilibrium for the first stage with \( q_1 \in [q^M, q^H] \) and \( q_2 \in [q^L, q^H] \). For the same reason we also conclude that there is no Nash equilibrium with \( q_1 \in [q^M, q^H] \) and \( q_2 \in [0, q^L] \). If \( q_1 \in [0, q^M] \) and \( q_2 \in [q^L, q^H] \), the optimal admission policy is given by \( (a_1^* = \hat{a}, a_2^* = \varphi(q_2)) \). We find that in this case university 2’s payoff function is strictly decreasing in its quality level. Consequently, there is no Nash equilibrium for the first stage with \( q_1 \in [0, q^M] \) and \( q_2 \in [q^L, q^H] \). Finally, given that \( q_1 \in [0, q^M] \) and \( q_2 \in [0, q^L] \), the optimal admission policy equals \( (a_1^* = \hat{a}, a_2^* = 0) \). In this case we
find the following asymmetric solution

\[ q_1^{*A} = \frac{2}{\gamma(4 - 3\eta)}, \quad q_2^{*A} = \frac{2}{\gamma(4 - \eta)}. \quad \tag{a30} \]

Second order conditions are always satisfied and the equilibrium obeys the assumptions we started from, namely \( q_1^{*A} \in [0, q^M] \) and \( q_2^{*A} \in [0, q^L] \).

We claim however, that this equilibrium (a30) is only local since the low quality university 2 always has an incentive to deviate to \( q_1^{*A} + \epsilon \).

Proof: Remark that when using solution (a30) we find that the payoff of university 1 always exceeds the payoff of university 2, \( U_1^{*A} > U_2^{*A} \). Hence, if university 2 deviates to \( q_1^{*A} + \epsilon \), it attracts all the high-ability students (those with \( a > \frac{1}{2} \)) who used to attend university 1 and consequently improves its payoff. See Figure 14. In this Appendix we assume that \( c = 0 \) but this argument can be extended for sufficiently small values of \( c > 0 \). To summarize, for very small values of the
mobility cost $c$ the asymmetric equilibrium only holds locally, because the low quality university has an incentive to surpass the high quality university. But, the higher the value of $c$, the higher the costs for the low quality university of surpassing the high quality university. See figure 15. For $c = 0.2$, university 2 still finds it optimal to jump to another area, i.e. to choose $q_1^* + \epsilon$. Hence, $q_2^*$ as defined in Theorem 4 is only a local best reply. For $c = 0.4$, however, $q_2^*$ becomes university 2’s global best reply. Notice the analogy with De Fraja and Iossa [6].

**Appendix 4: Values of $\alpha$ different from 2**

Consider the more general utility functions we introduced at the beginning of the paper

\[
\begin{align*}
  u_1 &= \xi + q_1 - \alpha(1-a)q_1 - cx, \\
  u_2 &= \xi + q_2 - \alpha(1-a)q_2 - c(1-x).
\end{align*}
\] (a31) (a32)

The resulting market boundary ($u_1 = u_2$) becomes \(^{16}\)

\[
\hat{x}(a) = \frac{(1-\alpha(1-a))(q_1 - q_2) + c}{2c}.
\] (a33)

It follows that

\[
\begin{align*}
  \hat{x}(a) &= 0 \iff a = 1 - \frac{1}{\alpha} - \frac{c}{\alpha(q_1 - q_2)} = \hat{a} \quad (a34) \\
  \hat{x}(a) &= 1 \iff a = 1 - \frac{1}{\alpha} + \frac{c}{\alpha(q_1 - q_2)} = \check{a}.
\end{align*}
\] (a35)

Let us define *vertical dominance* as the situation in which the high quality university 1 attracts all highest ability students, while the low quality university 2 attracts all lowest ability students, i.e. $\hat{a} > 0$ and $\check{a} < 1$. (Notice that vertical dominance does not occur for values of $\alpha < 1$.) Then we find

\(^{16}\)Notice that, if $\alpha$ equals 0, the market boundary becomes independent of ability. In other words, whether a student prefers university 1 to university 2 or vice versa, only depends on that student’s geographical location and not on her ability.
that

\[
\text{if there is vertical dominance } \Rightarrow \frac{\partial d_i}{\partial q_i} = 0, \forall \alpha, \quad (a36)
\]

\[
\text{if there is no vertical dominance and } \alpha < 2 \Rightarrow \frac{\partial d_i}{\partial q_i} > 0, \quad (a37)
\]

\[
\text{if there is no vertical dominance and } \alpha > 2 \Rightarrow \frac{\partial d_i}{\partial q_i} < 0, \quad (a38)
\]

\[
\text{if } \alpha = 2 \Rightarrow \frac{\partial d_i}{\partial q_i} = 0. \quad (a39)
\]

In the paper we assumed that \( \alpha = 2 \), and, depending on the other parameter values \( (c,s,\gamma,\eta) \), we could determine (I) a symmetric equilibrium, (II) two asymmetric equilibria, (III) a symmetric and two asymmetric equilibria, and (IV) no equilibrium. Now the question is whether this result can be extended to other values of \( \alpha \).

First, we find that the symmetric equilibrium analogous to Theorem 3 remains when \( \alpha \neq 2 \)

\[
q^*S = \frac{1}{2\gamma(6 + \alpha(3 + \eta) - 3\eta)}[6 - 3\alpha - 6\gamma(2 - \eta) + (9(-2 + \alpha - 2\gamma(-2 + \eta))^2 + 12\gamma(2s + \alpha(3 + \eta) - 3\eta)]^{1/2} \quad (a40)
\]

\[
a^*S = 0. \quad (a41)
\]

One important difference with the case \( \alpha = 2 \) has to be noted. For values of \( \alpha \neq 2 \) we do find an effect of a change in the per student allowance \( s \) on the symmetric equilibrium quality levels. More specifically, for values of \( \alpha < 2 \) we find that if the government increases the per student allowance \( s \), the universities will react by providing higher quality teaching, \( \frac{\partial q^*S}{\partial s} > 0 \). This stems from the fact that in this case an increase in quality increases their demand. See (a37). For values of \( \alpha > 2 \) the opposite holds.

Second, numerical experimentation learns that the asymmetric equilibrium analogous to Theorem 4 can exist for values of \( \alpha \neq 2 \). Unfortunately, we can not give a general expression. Remark that the asymmetric equilibrium can never exist for values of \( \alpha \leq 1 \) since this does not allow for vertical dominance. In other words, the effect of the effort cost on a student’s utility has to be strong enough \( (\alpha > 1) \) for vertical product differentiation to arise in equilibrium. Even for \( \alpha \neq 2 \) there is no influence of \( s \) on the asymmetric equilibrium quality levels. This stems from the fact that \( \frac{\partial d_i}{\partial q_i} = 0 \) in case of vertical dominance. For values of \( \alpha \neq 2 \) the sizes of the market shares of
the universities no longer equalize in the asymmetric equilibrium. The market share of the high
group university equals \( \frac{1}{2} \), while the market share of the low group university equals \( 1 - \frac{1}{2} = \frac{1}{2} \).
Hence, \( n_1^{*A} > n_2^{*A} \Leftrightarrow \alpha < 2 \) and \( n_1^{*A} < n_2^{*A} \Leftrightarrow \alpha > 2 \). Moreover, \( U_1^{*A} > U_2^{*A} \Leftrightarrow \alpha < 2 \) and \( U_1^{*A} < U_2^{*A} \Leftrightarrow \alpha > 2 \).

**Appendix 5: No per student allowance \((s = 0)\)**

If the budget of a university consists solely of the lump sum amount \( F \), a university’s payoff function equals

\[
U(q_i, q_j; a_i, a_j) = n_i q_i + \gamma \left[ F - (1 - \eta \bar{a}_i) n_i q_i^2 \right]. \tag{a42}
\]

If university \( i \) maximizes this payoff function with respect to \( a_i \), subject to no constraint on \( a_i \), the solution becomes

\[
a_i^* = \frac{-1 + \gamma q_i}{q_i \gamma \eta}. \tag{a43}
\]

Since we concluded in the paper that at the equilibria the symmetric and asymmetric quality levels
are not affected by the per student allowance \( s \), it is straightforward that they do not change
when we assume \( s = 0 \). However, the conditions characterizing the equilibria do change. For the
symmetric solution comparable to Theorem 3 we need

\[
c > \frac{\eta}{6\gamma(1 - \eta)} \quad \text{and} \quad c > \frac{6\eta}{5\gamma(-2 + \eta)^2}. \tag{a44}
\]

For the asymmetric solution comparable to Theorem 4 we need

\[
c < \frac{8\eta^2 \sqrt{4 - \frac{7n}{3}}}{\gamma(4 - 3\eta)^2(-4 + \eta)}, \quad c < \frac{24\eta}{5\gamma(-4 + \eta)(-4 + 3\eta)}, \quad c < \frac{2\sqrt{6} \sqrt{\frac{(-2 + \eta)^2}{(-4 + \eta)}}}{\gamma(4 - 3\eta)}
\]

\[
c < \frac{4 \left[ -3\gamma\eta^2(24 - 26\eta + 5\eta^2) + \sqrt{3} \gamma^2 \eta^2(8 - 6\eta + \eta^2)^2(16 - 28\eta + 15\eta^2) \right]}{\gamma^2(-4 + \eta)^2(16 - 28\eta - 8\eta^2 + 15\eta^2)} \tag{a45}
\]

Numerical experimentation teaches us that the conditions given in (a44) are never satisfied when
the conditions in (a45) are satisfied. In other words, when the universities are no longer partly
funded on a per student basis, the symmetric equilibrium and the two asymmetric equilibria can
never coexist. This confirms the findings of De Fraja and Iossa [6].
References


