Resonant enhancement of the FFLO state by a 1D optical potential

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Introduction
We study a new mechanism for superfluidity in ultracold Fermi gases with spin imbalance: the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state.

Ultracold Fermi gases
• Dilute gas of atoms cooled down to a few nanokelvin
• Fermions in two different states: spin-up & spin-down → form Cooper pairs → superfluid state
• Great control over system parameters: density, temperature, interaction strength, spin imbalance...
• Optical lattice: artificial crystals of light

Spin Imbalance
• Suppresses formation of Cooper pairs
• At critical polarization:
  superfluid → normal transition

The FFLO state
• BCS: pairs have zero momentum
• FFLO: pairs have nonzero momentum Q → translates Fermi surface of minority spins by Q

Method
Starting from the partition sum in path integral formulation we derive the free energy, which can be used to find the ground state of the system.

1) Partition sum of a 3D Fermi gas with spin imbalance
\[ Z = \int D\psi D\bar{\psi} \exp \left( -\sum_{k} \bar{\psi}_{k+} \left( -\epsilon_{k+} + \xi_{k+} \right) \psi_{k+} - \frac{g}{\beta \hbar} \sum_{\mathbf{k} \mathbf{k}_0} \bar{\psi}_{\mathbf{k}} \gamma^{\dagger} \gamma_{\mathbf{k}_0} \psi_{\mathbf{k}_0} \right) \]
- without 1D potential → \[ \xi_{k+} = k^2 + k_0^2 + \frac{\hbar^2}{m} Q^2 - \mu \]
- with 1D potential → \[ \xi_{k+} = k^2 + k_0^2 + \frac{\hbar^2}{m} Q^2 + \frac{\hbar^2}{m} (1 - \cos(k_0 / Q)) - \mu \]

2) Hubbard-Stratonovich transformation: 4th order → 2nd order interaction term
\[ \exp \left( -\sum_{\mathbf{q}} \bar{\psi}_{\mathbf{q}} \bar{\psi}_{-\mathbf{q}} \psi_{\mathbf{q}} \psi_{-\mathbf{q}} \right) = \int D\Delta D\Delta' \exp \left( \sum_{\mathbf{q}} \frac{\Delta^2}{g} + \Delta \bar{\psi}_{\mathbf{q}} \psi_{\mathbf{q}} + \Delta' \bar{\psi}_{-\mathbf{q}} \psi_{-\mathbf{q}} \right) \]

3) Saddle point approximation
\[ \Delta_0 = \sqrt{\frac{\hbar^2}{m} \frac{\beta \hbar g}{\Delta^2} (\mathbf{q} - \mathbf{Q})} \Delta \rightarrow \text{FFLO state is allowed: } \mathbf{Q} \neq 0 \]

4) Remaining fermionic path integral is quadratic and can be solved exactly
\[ \int D\bar{\psi} D\psi \exp \left( -\sum_{\mathbf{k} \mathbf{k}_0} \bar{\psi}_{\mathbf{k}} \left( \epsilon_{\mathbf{k}} + \xi_{\mathbf{k}} \right) \psi_{\mathbf{k}_0} \right) = \det \left( A_{\mathbf{k} \mathbf{k}_0} \right) \]

5) Free energy
\[ \Omega_{\mathbf{q}}^{\text{FFLO}} = \frac{g}{\beta \hbar} \sum_{\mathbf{k}} \bar{\psi}_{\mathbf{k}} \gamma^{\dagger} \gamma_{\mathbf{k}} \psi_{\mathbf{k}} \]

Constructing the phase diagrams
Gap equation
\[ \frac{\partial \Omega_{\mathbf{q}}^{\text{FFLO}}}{\partial \mathbf{q}} = 0 \]
Number equations
\[ \frac{\partial \Omega_{\mathbf{q}}^{\text{FFLO}}}{\partial \mu} = n, \quad \frac{\partial \Omega_{\mathbf{q}}^{\text{FFLO}}}{\partial \zeta} = \delta n \]

Results
We construct the phase diagrams of a spin-imbalanced 3D Fermi gas with and without a 1D optical potential and study the effects of the potential on the FFLO state.

I. Phase diagram at fixed density

II. Phase diagram at fixed scattering length

• Transformation: (\mu, Q)-phase diagram (A) → (1/k0, \beta)-phase diagram (B)
• Both BCS- and FFLO pairing become suppressed above a certain interaction strength (red circle and red square in (B))
• Edge of the Brillouin zone hinders the formation of Cooper pairs

Conclusion
By adding a 1D optical potential, the presence of the FFLO state in the phase diagrams of a spin-imbalanced 3D Fermi gas is substantially enlarged. This enhancement is optimal when the wave vector of the FFLO state becomes equal to the wave vector of the laser.

References:
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