Short-pulse laser absorption in very steep plasma density gradients

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(Received 23 May 2006; accepted 21 August 2006; published online 15 September 2006)

An analytical fluid model for resonance absorption during the oblique incidence by femtosecond laser pulses on a small-scale-length density plasma \([k_0L \ll (0.1, 10)]\) is proposed. The physics of resonance absorption is analyzed more clearly as we separate the electric field into an electromagnetic part and an electrostatic part. It is found that the characteristics of the physical quantities (fractional absorption, optimum angle, etc.) in a small-scale-length plasma are quite different from the predictions of classical theory. Absorption processes are generally dependent on the density scale length. For shorter scale length or higher laser intensity, vacuum heating tends to be dominant. It is shown that the electrons being pulled out and then returned to the plasma at the interface layer by the wave field can lead to a phenomenon like wave breaking. This can lead to heating of the plasma at the expense of the wave energy. It is found that the optimum angle is independent of the laser intensity while the absorption rate increases with the laser intensity, and the absorption rate can reach as high as 25\%. © 2006 American Institute of Physics.

[DOI: 10.1063/1.2354583]

The interaction of ultrashort laser pulses (\(\tau_p < 1\) ps) with solid targets has become an important field of study because of many applications, such as the fast ignition scheme of inertia confinement fusion,\(^1\) the plasma-based particle accelerator,\(^2\) coherent x/\(\gamma\)-ray sources,\(^3\) etc. For most of these applications, the nature of the absorption process must be determined. As we known, the density scale length of the plasmas generated from the target surfaces can be estimated as

\[
L = (\partial \ln n_f/\partial x)^{-1} = c_s/\tau_p,
\]

where \(c_s = \sqrt{(ZT_e + T_i)/M_i}\) is the ion sound speed and \(\tau_p\) is the pulse duration. For a pulse duration \(\tau_p = 400\) fs, laser wavelength \(\lambda_0 = 1.05\) \(\mu\)m, and electron temperature \(T_e = 1\) keV, the scale length is \(L \approx 0.057\lambda_0 \approx 0.4c/\omega_p\). In this case, collisional absorption is highly insufficient and resonance absorption at the critical surface is suggested to be one of the major absorption mechanisms.\(^5\)

Resonance absorption is the collisionless absorption of \(p\)-polarized radiation in a plasma with a finite scale length. Some experiments\(^6,7\) show that it plays an important role even for plasmas with a scale length considerably shorter than the laser wavelength. However, to our knowledge, the previous theoretical works\(^4,5\) on resonance absorption are only valid for the case in which \(L > \lambda_0\). For example, the classical Ginzburg function\(^4,5,8\) gives the peak value of the absorption at \((k_0L)^{1/3}\) sin \(\theta_0 = 0.8\), but evaluating for the case \(k_0L = 0.5\) gives the optimum angle \(\theta_{opt} = 90^\circ\), which is unphysical. Therefore, it is worthwhile to reexamine this absorption mechanism in a sharp density gradient plasma.

At higher laser intensity, say \(L < \omega_p/\omega_0\), the electrons being pulled out and then returned to the plasma at the interface layer by the wave field can lead to a phenomenon like wave breaking. Thus, the electron plasma wave is hard to develop and vacuum heating\(^9\) tends to be dominant. Here, \(\omega_p = eE_0/m_e\omega_0\) is the quiver velocity of the electron, \(E_0\) is the electric field normal to the interface, and \(\omega_0\) is the laser frequency. A number of theoretical and numerical studies have added to this important mechanism.\(^9-12\) However, this topic is still worthy of further investigation because there are still some points that the previous theoretical works do not solve.\(^9,10\) For example, why is the optimum angle about \(45^\circ\)?\(^11\) Furthermore, with a slightly surface expansion of scale length, the laser field would pull more electrons into the vacuum, and thus be even more strongly absorbed.\(^11\) In this Brief Communication, a new detailed theoretical work for vacuum heating is also presented. It is found that the absorption rate increases with \(L\) as long as \(L\) does not significantly exceed the electron excursion length, and the optimum angle is independent of the laser intensity.

We consider a \(p\)-polarized plane electromagnetic wave incident at angle \(\theta_0\) onto a plasma slab with electron density \(n_e(z)\). Without loss of generality, we take the electric vector in the \(x, z\) plane and the magnetic vector in \(e_y\). Hence the electromagnetic fields have the form
The first term is the incident laser wave, and the second term is dominant since the electric field can drive a large freely outgoing waves at the vacuum side and evanescent waves associated with the incident laser wave while the imaginary part determines the damping of the reflected wave. From Maxwell’s equations, we can get an equation for the magnetic field $B_y$ easily,

$$\frac{d^2 B_y(z)}{dz^2} - \frac{1}{e_1} \frac{dB_y(z)}{dz} + k_0^2 (e_1 - \sin^2 \theta_0) B_y(z) = 0,$$

where $e_1$ is the dielectric function. For a cold plasma, $e_1(z) = 1 - \omega_p^2(z)/\omega_0^2(1 + \nu/\omega_0)$, where $\omega_p^2 = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency. $\nu$ is a small effective damping rate ($\nu/\omega_0 \ll 1$). The boundary conditions for Eq. (2) are freely outgoing waves at the vacuum side and evanescent waves at the high-density side. Therefore, by assuming that the plasma density is a linear function of position $n_z = n_{z=0} z/L$ from $z=0$ to $z=LN$, followed by a plateau at density $n_{z=LN}$ (here $n_{z=0} = m_e c^2 / 4 \pi e^2$ is the critical density and $N=15$), Eq. (2) can be solved numerically.

On the other hand, $E_z = -\bar{\partial} A_z / c - \bar{\partial}_z \phi$, where $A_z$ is the vector potential and $\phi$ is the scalar potential. Therefore, we can separate the electric field into two parts: $E_z = E_{z,0} + E_{z,1}$, where $E_{z,0} = -\bar{\partial} A_z / c$ is the electromagnetic component associated with the incident laser wave while $E_{z,1} = -\bar{\partial}_z \phi$ is the electrostatic component associated with an electron plasma wave. It is important to stress that in the present question the latter is dominant since the electric field can drive a large plasma wave at the turning point resonantly. Starting from $B = \nabla \times A$, $\nabla \cdot A = 0$, and $E_{z,1} = -\bar{\partial} A_z / c$, we obtain

$$\frac{d^2 E_{z,1}}{dz^2} - k_0^2 \sin^2 \theta_0 E_{z,1} = -k_0^2 \sin \theta_0 B_y.$$

Here $B_y$ is given by Eq. (2). Together with the boundary conditions, $E_{z,0}$ can be solved from Eq. (3). Combining Eq. (1) and Ampere’s law, we have $E_z = \sin \theta_0 B_y / E_{z,0}(z)$. Therefore, we obtain the absorption rate $f_{RA}$ due to the damping of the plasma wave $E_{z,1}$,

$$f_{RA} = \frac{\nu}{8 \pi I_x} \int_{-\infty}^{\infty} |E_{z,1}|^2 dz,$$

where $I_x = c E_{z,0}^2 / 8 \pi$ is the incident power. Physically, an electron plasma wave is resonantly excited near the critical density. When this plasma wave is damped, the energy is transferred to the plasma. From this point, our new model more precisely emphasizes the physics of resonance absorption and captures its basic features. It should be mentioned that as long as the damping rate $\nu$ is controlled within a certain range ($\nu < 0.05 \omega_0$), the fractional energy absorbed is nearly independent of this damping rate. The reason is that the electrostatic component $E_{z,1}$ is negatively related to $\nu$: as the damping becomes larger, the plasma wave develops more difficulty.

Equations (2) and (3) have been solved numerically. The spatial variations of the electromagnetic fields $B_y$, $E_{z,0}$, and $E_{z,1}$ are plotted in Fig. 1. The absorption is caused by the nonzero wave field $E_{z,0}$, which resonantly drives a large plasma wave $E_{z,1}$ with frequency $\omega_p$. The classical fractional absorption is dependent on $\tau = (k_0 L)^{1/3} \sin \theta_0$ for $k_0 L \gg 10$ is reproduced; see Fig. 2. However, as the scale length is reduced below $k_0 L < 10$, the absorption peaks at smaller $\tau$.

Here we compare our numerical results and the classical results in detail; see Fig. 3(a). In a large scale length ($k_0 L \gg 10$) plasma, our numerical results are in good agreement with the classical results, satisfying $(k_0 L)^{1/3} \sin \theta_0 = 0.8$. But in a sharp density gradient ($k_0 L \ll 1$), the numerical results strongly disagree with the classical results, which are unsuitable in this region. Figure 3(b) shows more clearly under the logarithm coordinate. We can easily see that the optimum angle is still about 45° when $k_0 L = 0.1 (L = 0.016 \lambda_d)$ in our results, while it is greater than 90° when $k_0 L \approx 0.5$ in the classical theory. Fitting the numerical results, we obtain

![Graph](Image URL)
\[(k \omega_0)^{0.6} + 1.97] (\sin \theta_0 - 0.09) = 1.42. \] (5)

The squares in Fig. 3(b) are the peak power absorption rate under a different scale length. We can see that the power absorption rate decreases with the decreasing of the scale length. Note that the minimum scale length we consider for resonance absorption is \(k_0 L = 0.1\), but at higher laser intensity, resulting in \(L < \frac{v_0}{\omega_0}\), vacuum heating will play a more important role.\(^6\), \(^9\)\(^-\)\(^12\)

Vacuum heating is an effect related to the classical resonance absorption\(^12\) in that the laser electric field drives the electrons across a density gradient. An important difference between them is that in the latter process, the scale length is long enough; the electric field can resonantly excite a large electrostatic plasma wave at the critical point. In vacuum heating, on the other hand, the density gradient scale length is much smaller (\(L < \frac{v_0}{\omega_0}\)), so that no such resonance exists, and the electron motion is mainly driven by the electric field of the laser. In fact, the wave breaking phenomenon occurs within about one laser period,\(^9\),\(^10\) so the electrostatic wave is hard to develop. Therefore, the electromagnetic component is dominant, i.e., \(u \gg v\). Here \(u = eA_z/mc\) is the velocity associated with the electromagnetic component while \(v = e\phi/m\omega_0\) is the velocity associated with the electrostatic component. Therefore, we can safely neglect the electrostatic component in the wave equation for the laser radiation. In order to compare vacuum heating with resonance absorption in a small-scale-length plasma, we use the same model as mentioned above. From Maxwell’s equations, we have the equation for the longitudinal part (parallel to \(e_z\)) of the laser radiation,

\[ \frac{d^2A_z}{dz^2} + k_0^2(e_1 - \sin^2 \theta_0)A_z = 0, \] (6)

where \(e_1 = 1 - \omega_p^2 / \omega_0^2\) and the density profile is of the type \(n_e = n_{te} e^{-z/L}\) from \(0 \leq z = -LN\), followed by a plateau at density \(Nn_{te}\). Therefore, there are three different regions in this model: (i) vacuum, (ii) linear density layer, and (iii) plateau.

The solutions of Eq. (6) are (i) \(A_z = A_0 \sin \theta_0 \sin(k_0 \cos \theta_0 z + \phi)\), (ii) \(A_z = A_0 [\alpha \text{Ai}(\eta) + \beta \text{Bi}(\eta)]\), and (iii) \(A_z = \alpha' \exp(-\kappa k_0 z)\), separately. Here \(\eta = (\omega_0^2 / c^2 L)^{1/3} (z - L \cos^2 \theta_0)\) and \(\kappa = (N - \cos^2 \theta_0)\) are used, and \(\text{Ai}(\eta)\) and \(\text{Bi}(\eta)\) are the well-documented Airy functions. With the knowledge of the theory of classical electrodynamics, we know that the boundary conditions are \(\mathbf{e}_\perp \times (\mathbf{B} - \mathbf{B'}) = 0\) and \(\mathbf{e}_\perp \times (\mathbf{E} - \mathbf{E'}) = 0\). The electromagnetic fields are obtained from Maxwell’s equations,

\[ E_z = (\omega_0/k_0 \sin \theta_0) \partial_z A_z, \quad E_y = -i(k_0 / \sin \theta_0) e_y A_z. \]

Therefore, the boundary conditions give the unknown constants,

\[ \alpha = \sin \theta_0 / \sqrt{[\text{Ai}(\eta_0) + c_1 \text{Bi}(\eta_0)]^2 - [\text{Ai}'(\eta_0) + c_1 \text{Bi}'(\eta_0)]^2 / \eta_0}, \quad \beta = c_1 \alpha, \]

where \(c_1 = (|k| \omega_0^2 / c^2 L)^{1/3} \text{Ai}(\eta_0) + \text{Ai}'(\eta_0) + \text{Bi}(\eta_0) + \text{Bi}'(\eta_0)) \eta_0 = \eta|_{z = 0}.\)

It is important to stress that, in the limit \(L = 0\), our model recovers the step function model discussed by Kato et al.\(^10\)

The spatial variation of \(A_z\) is plotted in Fig. 4. We can see

\[ \frac{d^2A_z}{dz^2} + k_0^2(e_1 - \sin^2 \theta_0)A_z = 0, \] (6)
that $A_z(0^+)$ decreases with the decreasing of the scale length. When $L=0$, a discontinuous point appears at $z=0$.

As we know, in such a sharp density gradient, the electromagnetic fields drop rapidly into the plasma; see Fig. 4. Therefore, only the electrons in the skin layer (the absorption layer) make a contribution to the absorption. \cite{10} According to Gibbon’s theory,\cite{11} the thickness of the absorption layer $z_d$ can be defined as the effective excursion length of the electron at the interface, $z_d=2u_t/\omega_0$, where $u_t=(eA_0/m_e)c\times[\alpha A_i(\eta_0)+\beta B_i(\eta_0)]$. Notice that the thickness of the linear density layer is $D_L=LN$. Therefore, in such a linear density profile with a plateau, the number of electrons in the absorption layer can be obtained by $N_a=\int_0^{z_d}n_a(z)dz$. If $L \geq z_d/N$, we obtain $N_a=\varepsilon^2_{pl}/2L$; if the scale length is much shorter, $L < z_d/N$, we obtain $N_a=N_{n_0}z_d-L^2n_{cr}/2$. Thus, the energy absorbed from the laser light by the electrons is just $W_{abs}=(1/2)\mu U^2_nN_a$. It is also equal to the fraction of power, $P_{abs}=P_{abs}/P_L$, lost from the electromagnetic wave, i.e.,

$$f_{VH}=\begin{cases} 8\varepsilon^2_{pl}[\alpha A_i(\eta_0)+\beta B_i(\eta_0)]^4/k_0L, \\ 4[\alpha A_i(\eta_0)+\beta B_i(\eta_0)]^2[2\varepsilon_{pl}N(\alpha A_i(\eta_0)+\beta B_i(\eta_0)) - k_0LN^2/2], \end{cases} L > z_d/L,$$

$$L \leq z_d/L.$$

(7)

Here $\varepsilon_{pl}=eA_0/m_e c^2$ is the normalized amplitude of the vector potential. In the overdense region, the density can be as high as tens of critical density, therefore $L \geq z_d/N$ is satisfied in the range of the scale length in which we are interested here (0.01 $\leq k_0L < 0.1$). Notice that as long as $NL > 0.1k_0^{-1}$ (e.g., $k_0L > 0.01$ when $N \approx 10$), we have $B=0$. Therefore, in such a case, the optimum angle $\theta_{opt}$ can be obtained by $\partial f_{VH}/\partial \theta_0=0$, i.e.,

$$\tilde{A}_i(\tilde{\eta}_0) + 2\varepsilon_{pl}^2\tilde{A}_i^2(\tilde{\eta}_0) = -\frac{\varepsilon_{pl}}{\eta_0^4\tilde{\eta}_0^4} \tilde{A}_i(\tilde{\eta}_0)\tilde{A}_i^2(\tilde{\eta}_0).$$

Equation (8) describes the relationship between $\theta_{opt}$ and $L$ since $\tilde{\eta}_0=-L^{2/3}\cos \theta_{opt}$ and $\tilde{\eta}=L^{2/3}\sin \theta_{opt}$. Notice that $\theta_{opt}$ is independent of the amplitude of the incident laser. In the nonrelativistic regime, $\theta_{opt} = 45^\circ$, consistent with the experimental\cite{13,14} and simulation results.\cite{11} The absorption rate is sensitive to the scale length and incident angle; see Fig. 5. It is found that the power absorption rate increases with $L$ as long as $L \leq v_{os}/\omega_0$, a result that agrees with Gibbon’s simulation.\cite{11} It is also found that the power absorption rate reaches as high as 25% when $k_0L \approx 0.1$.

In conclusion, in a sharp density gradient, absorption mechanisms, including resonance absorption and vacuum heating, are investigated theoretically in order to explore the interaction of femtosecond laser pulses with solid targets. It is found that in the intensity regime from $10^{15}$ to $10^{17}$ W $\mu$m$^{-2}$/cm$^{-2}$, resonance absorption dominates when the scale length $k_0L > 0.1$ while vacuum heating dominates when $k_0L < 0.1$. It is also found that when $k_0L \approx 0.1$, both resonance absorption and vacuum heating peak around $45^\circ$. Furthermore, we have found that the absorption rate due to vacuum heating increases with $L$ as long as $L \leq v_{os}/\omega_0$.

This work was supported in part by the National Hi-Tech Inertial Confinement Fusion (ICF) Committee of China, National Science Foundation of China (NSFC) Grants No. 10135010, No. 10375011, No. 10335020/A0506, No. 10474081, and No. 10576035, the Natural Science Foundation of Shanghai (Project 05ZR14159), the Special Funds for Major State Basic Research Projects of China, and the Science Foundation of CAEP. One of the authors (Z.Y.C.) would like to thank the Bilateral project between Flanders and China.

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